Liquid Water Content (LWC) Sensor Equations

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January 2017

LWC Equations.

The following discussion pertains to Anasphere's liquid water content (LWC) sensor, which measures the above-freezing liquid water content of clouds. Key user-provided parameters include the median volume diameter of droplets, air velocity, and ambient meteorological parameters.

A separate discussion pertains to the related supercooled liquid water content (SLWC) sensor; do not use the following math for that sensor. However, the SLWC math documents may provide the reader with additional useful reference information.

The coated LWC wire sensor is modeled as shown in Figure 1. The coating is modeled as a cylinder that expands as it absorbs water from the surrounding air with length L_g and diameter D_g. Here, the subscript g indicates the coating, which becomes gel-like after absorbing water. To find an equation for the LWC, the equation for a wire sweeping through a volume V of air and collecting a percentage ε of the liquid water is again used:

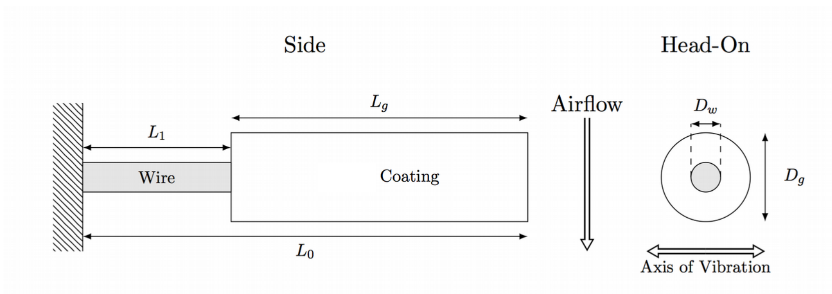

Figure 1. Wire loaded with coating.

$$LWC = \frac{\Delta M_g}{\epsilon V}. \tag{1}$$

M_g is the sum of the dry coating's mass and any water mass accumulated. Since the diameter of the coating grows as it accumulates water, the volume cannot simply be modeled as a rectangular box like it was for the icing-wire. Instead, the volume is approximated as a wedge-like shape as shown in Figure 2.

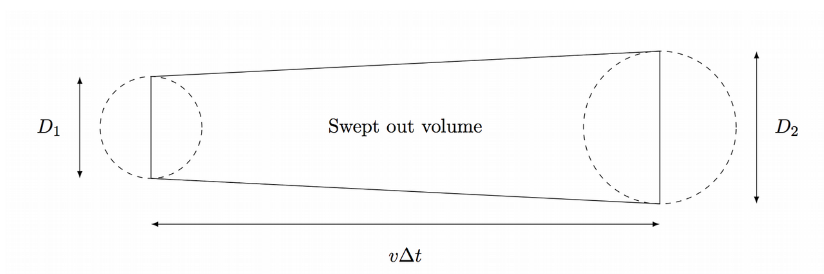

Figure 2. Swept volume approximation.

With this, the volume becomes

$$V=\frac{1}{2}v\Delta t(D_2+D_1)L_g, \quad (2)$$

where D_1 is diameter of the coating from the previous frequency measurement and D_2 is the diameter of the coating at the current measurement. Using this, (1) becomes

$$\begin{aligned} LWC &= \frac{\Delta M_g}{\frac{1}{2}\epsilon v\Delta t(D_2+D_1)L_g}, \\ &= \frac{2}{\epsilon v(D_2+D_1)}\frac{\Delta\lambda_g}{\Delta t}, \\ &= \frac{2}{\epsilon v(D_2+D_1)}\dot{\lambda}_g, \end{aligned} \quad (3)$$

where λ_g is the linear density of the gel, which includes the water. This time, both the linear density and the two diameters are functions of the wire's frequency. To find the linear density's time derivative, (10) is used with the appropriate variable substitutions:

$$\begin{aligned} f^2 &= \frac{\pi^2 E_w\left(I_w+\frac{cI_g}{\beta_g}\right)}{128L_0^4(a\lambda_w+b\lambda_g)}, \\ f^2 &= \frac{af_0^2\lambda_w\left(I_w+\frac{cI_g}{\beta_g}\right)}{I_w(a\lambda_w+b\lambda_g)}. \end{aligned} \quad (4)$$

Here,

$$I_g=\frac{\pi}{64}(D_g^4-D_w^4) \quad (5)$$

and

$$\beta_g=\frac{E_w}{E_g}. \quad (6)$$

Next, the gel's stiffness is assumed to be negligible, which means that β_g approaches infinity. Taking the limit of (4) as β_g approaches infinity yields

$$f^2=\frac{a\lambda_w f_0^2}{a\lambda_w+b\lambda_g}. \quad (7)$$

λ_g and its time-derivative are then

$$\lambda_g=b_0\left(\frac{f_0^2}{f^2}-1\right) \quad (8)$$

and

$$\dot{\lambda}_g = \frac{-2 b_0 f_0^2}{f^3} \dot{f} . \qquad (9)$$

The gel's diameter was experimentally determined to have a linear relationship with λ_g. The equation for D_g was found to be

$$D_g = 0.2793 \lambda_g + 0.0014 \qquad (10)$$

using SI units.

Using all of this, (3) becomes

$$LWC_i = \frac{-4 b_0 f_0^2}{\epsilon v [D_g(f_i) + D_g(f_{i-1})] f_i^3} \dot{f}_i , \qquad (11)$$

where i indicates the current measurement and i-1 indicates the previous measurement.

<u>Implementation of LWC Equations.</u>

One thing to note about the above equation for LWC is that it is very sensitive to noise and so smoothing of the input data becomes very important. For this project, a Savitzky–Golay filter with a window of 15 points was applied to all of the experimental data before applying the equation. This proved effective at eliminating higher frequency noise.

The frequency time-derivative was calculated using the symmetric difference method. This is given by

$$\frac{df(x_i)}{dx} = \frac{f(x_{i+1}) - f(x_{i-1})}{\Delta x} , \qquad (12)$$

where the interval between measurements is assumed to be constant. This method is more accurate than a simple finite difference but causes a loss of a point at either end of the data since no derivative can be calculated at the edges.

One problem with (11) is that the collection efficiency equation assumes a constant diameter over the course of the measurement interval. This is clearly at odds with the model used for the LWC sensor. For this implementation, a simple average of the two diameters in (11) was used to calculate the efficiency.

Below is a table listing the values used for constants in the equations above.

Constant	**Value**
f_0	44.2 Hz

L_0	9.4×10^{-2} m
L_1	3.2×10^{-2} m
L_g	6.2×10^{-2} m
λ_w	2.22×10^{-2} kg/m
D_w	6.096×10^{-4} m
E_w	2.0×10^{11} Pa